ABSTRACT

The Texas A&M water channel experiment is modified to examine the effect of single-mode initial conditions on the development of buoyancy-driven mixing (Rayleigh-Taylor) with small density differences (low-Atwood number). Two separated stratified streams of $\sim 5^\circ C$ difference are convected and unified at the end of a splitter plate outfitted with a servo-controlled flapper. The top (cold) stream is dyed with Nigrosine and density is measured optically through the Beer-Lambert law. Quantification of the subtle differences between different initial conditions required the optical measurement uncertainties to be significantly reduced. Modifications include a near-uniform backlighting provided through quality, repeatable, professional studio flashes impinging on a white-diffusive surface. Also, a black, absorptive shroud isolates the experiment and the optical path from reflections. Furthermore, only the red channel is used in the Nikon D90 CCD camera where Nigrosine optical scattering is lower. This new optical setup results in less than 1% uncertainty in density measurements, and 2.5% uncertainty in convective velocity. With the Atwood uncertainty reduced to 4% using a densitometer, the overall mixing height and time uncertainty was reduced to 5% and 3.5%, respectively. Initial single-mode wavelengths of 2, 3, 4, 6, and 8 cm were examined as well as the baseline case where no perturbations were imposed. All non-baseline cases commence with a constant velocity which then slows, eventually approaching the baseline case. Larger wavelengths grow faster, as well as homogenize the flow at a faster rate. The mixing width growth rates were shown to be dependent on initial conditions, slightly outside of experimental uncertainty.

INTRODUCTION

The presence of the inherently complex and chaotic fluid motions of turbulence enable efficient mixing yet hinder predictability. This is a challenge as turbulent mixing is of interest to many fields of science and engineering, such as momentum mixing (also known as drag) [1], temperature mixing in heat transfer [2], and species mixing of dissimilar fluids (including multiphase flows) [3]. In 1941 Kolmogorov hypothesized that all turbulent flows achieve a self-similar state as long as dissipation is homogeneous, isotropic, and in equilibrium with turbulent production [4]. This self-similarity hypothesis lead to many useful principles of turbulence such as energy transfer [5], active and inactive motions [6], and memory loss of initial conditions [7] that has well served the engineering and science communities. However, in some turbulent flows subtle deviations from Kolmogorov’s assumptions yield non-trivial phenomena. Examples include the very large scale structures found in boundary layer flows [8], intermittency in statistically homogenous and isotropic turbulence [9], and initial perturbation dependency in buoyancy driven turbulence [10].

Specifically in buoyancy driven turbulence, the assumption of memory loss of initial conditions leads to phenomenological conclusions regarding the growth rate models of the mixing layer between two fluids of dissimilar density [11, 12]. However, recent data show that the effect of the initial conditions are not lost for early time, and so the possibility exists to increase or decrease the rate at which mixing occurs [10, 13, 14]. Olson and Jacobs [14] experimentally verified that growth rates vary with short wavelength initial conditions, using a rocket rig with initial
conditions imposed by internal waves generated by vertical tank oscillations.

We hypothesize that the mixing height growth rate dependency on initial conditions exists also in the low-density difference regime. If correct, then fine-scale initial condition control and greater experimentally accessibility of the Texas A&M water channel apparatus will allow the physics to be explored in further detail. The challenge in proving this hypothesis is that the differences between initial conditions are subtle and thus requires a significant reduction of the mixing height uncertainty and time uncertainty from the optical measurements of density and velocity, which is the topic of the presented work.

BACKGROUND

Buoyancy driven turbulence, specifically as a heavy fluid residing over a lighter fluid, was first studied by Lord Rayleigh in 1883 [15]. In 1950 Taylor broadened this to include a heavy fluid that is accelerated into a lighter fluid [16]. Thus the title now bears both their names: the Rayleigh-Taylor hydrodynamic instability (RT). The RT instability is attributed to many natural occurring phenomena, including supernovae [17], nebula fingers, oceanic and atmospheric instabilities [18], magma diapers [19], and salt dome formations. In addition, the RT instability is partially responsible for the failure to attain a thermonuclear ignition in Inertial Confinement Fusion (ICF) [20], of particular interest to the present authors.

The parameter of importance in RT mixing is the Atwood number,

\[ A_t = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}, \quad (1) \]

a non-dimensionalization of initial density contrast between heavy and light fluids (\(\rho_1 > \rho_2\)). The limits on the \(A_t\) number are zero and one, with \(A_t = 1\) being the asymptotic limit where \(\rho_1\) is infinitely larger than \(\rho_2\) and \(A_t = 0\) where the two fluids have identical densities (e.g. are the same fluid). Through experimentation and numerical simulation it has been observed that for a small \(A_t\) number the RT instability will behave in a symmetrical fashion with “bubble” (rising fluid) and “spike” (falling fluid) formations growing qualitatively equally in size but in opposing directions (see Fig. 1). However, as the \(A_t\) number increases, the symmetrical nature of the mixing decreases [21]. In ICF the \(A_t\) number is about 0.9 deriving from a spherical capsule composed of a cryogenic solid Deuterium-Tritium (DT) shell that is imploded by laser surface ablation [22]. The outward ablated material generates an opposing force thus imploding the shell onto itself ultimately reaching a stagnation point where a hotspot is formed leading to thermonuclear ignition. The hydrodynamical instability occurs during the ablation process as the surface of the DT shell transforms into an expanding light density plasma that encapsulates a denser imploding DT material. In the accelerating reference frame, the high density plasma is driven outwards into the low density plasma which causes mixing between the two states of DT material. This results in internal shell deformation, potential shell rupture, and non-ignition [23].

The RT instability manifests itself in the unstable condition \(\nabla \rho \cdot \nabla P < 0\), where \(\rho\) and \(P\) are the fluid density and pressure, respectively [16]. Due to noise, natural or imposed, a perturbation is always guaranteed at the interface and thus initiates the instability, with baroclinic vorticity \((\nabla P \times \nabla \rho)/\rho^2\) along the interface as depicted in Fig. 1 [24]. Vorticity increases the amplitude of the interface and leads to the formation of familiar bubble and spike structures. As the bubble and spike formations move in opposite directions, shear generates vorticity along the interface leading to vortices (Kelvin-Helmholtz instability) which increase the surface area between the two fluids, inducing further mixing. The average height difference forming between the bubble and spike formations is known as the mixing width (or height),

\[ h = \frac{(h_b - h_s)}{2} \quad (2) \]

based on the non-dimensional sample-averaged (angle brackets) and span-averaged (overline) density

\[ \langle f_1 \rangle = \frac{(\langle \rho \rangle - \rho_2)}{(\rho_1 - \rho_2)} \quad (3) \]

where the bubble height \(h_b\) is defined at the \(\langle f_1 \rangle = 0.95\) threshold and spike height \(h_s\) at the \(\langle f_1 \rangle = 0.05\) threshold.

Over the last 25 years the temporal evolution of the mixing width has been characterized into three regimes: [7]

1. Exponential Growth, where any infinitesimal perturbation grows exponentially.

2. Saturation, where growth becomes non-linear and slows, large structures appear, and effect of initial conditions begin to subside.

3. Similarity, where viscosity effects are negligible on the growth rate, and the mixing is described by a similarity solution proportional to \(g t^2\).

The first regime describes the exponential growth of all infinitesimal perturbations where the most unstable wavelength

\[ \lambda_{\text{ms}} \approx 4 \pi \left( \frac{v^2}{g A_t} \right)^{\frac{1}{3}} \quad (4) \]

will grow to become the dominant wavelength [25]. This wavelength will be of import in presentation and discussion of the results.
The second regime characterizes the evolution of large structures that arise from the non-linear interactions of smaller structures (not necessarily from initial conditions). The evolution of large structures, as previously mentioned, is a combination of the bubble and spike formations with the secondary Kelvin-Helmholtz shear instability structures. These structures grow until they reach terminal velocity based upon their size [26]. The point at which terminal velocity is achieved is known as saturation, and the fully developed structures appear. From this point forward the transition to turbulence begins.

The third regime is denoted by similarity, and is defined by the Reynolds number growing to infinity. The Reynolds number \( \text{Re}=h \cdot h/\nu \) is characterized by the mixing width \( h \) (as defined in equation 2), its growth rate \( \dot{h} \), and viscosity \( \nu \). Self-similarity exists if the following three assumptions are made: \( \text{Re} \rightarrow \infty \), initial conditions are lost due to turbulence, and no other length scales of motion (including turbulent motions) are present. This results in \( gr^2 \) being the sole length scale for which to non-dimensionalize the mixing width \( h/\text{gr}^2 = \text{constant} = \alpha A_t \).

Our interest in this paper is the mode development during stages 1 (exponential growth) and 2 (saturation). Stage 3 has been extensively explored in previous work at the Texas A&M water channel [21, 27–29]. By sacrificing the higher \( A_t \) pertinent to ICF, the Texas A&M low-Atwood water channel facility achieves greater experimental accessibility, including high resolution density measurements, and thus provides a good platform in which to investigate the effect of initial conditions. The modifications to the experiment for implementing custom wavelength designs into the interface are discussed in the next section. In the optical measurements section, the setup and modifications made to improve the accuracy of the optical density measurements are discussed. How the newly reduced density uncertainty propagates to the mixing height uncertainty, along with the other uncertainties associated with the experiment, are discussed in the uncertainty analysis section. With the reduced overall mixing height uncertainty, the effect of different single-wavelength initial conditions on mixing growth rates are now observable and quantifiable and are reported in the results section.

**EXPERIMENTAL SETUP**

The low-Atwood number water channel at Texas A&M provides an accessible and relatively inexpensive experiment to accurately measure RT mixing. The present channel is constructed out of plexiglass and provides a test section which is 125 cm long, 32 cm tall by 20 cm wide as shown in Fig. 2 (left). The channel is partitioned along the horizontal axis into top and bottom sections by a splitter plate. Water from two separated 500 gallon tanks is pumped through individual pumps into the channel via an inlet plenum. The inlet plenum is exposed to atmospheric pressure and directs water to the top and bottom sections of the channel respectively. Flow laminators composed of an array of 5 mm diameter by 20 cm long straws are installed past the inlet plenum in order to minimize free stream turbulence. The flow exits through a plenum that is also exposed to atmospheric pressure. The exit plenum’s outflow is set to ensure uniform flow through the test section. Utilizing Taylor’s frozen-turbulence hypothesis [29], distance downstream translates to time by \( t = x/U \), where \( x \) indicates the axial distance downstream in the channel, and \( U \) is the uniform convective velocity.

A low-Atwood number \( A_t \sim 10^{-3} \) is attained by a slight temperature difference between the top and bottom flows in the channel (\( \sim 5.0^\circ \text{C} \) higher on the bottom). Water samples are drawn from both the cold and hot water tanks and analyzed with the use of a densitometer (Rudolph, Inc DDM2911).

A hinged foil was added to the channel at the end of the splitter plate, known as the flapper. The flapper spans the width of the channel with a negligible gap between channel walls and flapper. The flapper is 5.1 cm long and 6.4 mm thick at its thickest point located at the hinge section as shown in Fig. 2. The flapper tapers off to a 0.15 mm knife edge with a 41.8 cm radius of curvature.
and 3.6 degrees slope, which is sufficiently small to prevent flow separation over the flapper according to a triple deck boundary layer analysis [30]. The hinge design is based on a piano style hinge and provides a smooth transition between the splitter plate and the flapper to reduce possible flow disturbances. Aluminum was chosen for the flapper material to ensure minimal deflection of the flapper from the flow while in operation. The aluminum flapper was manufactured utilizing Computer Numeric Control Electrical Discharge Machining. This type of manufacturing ensures that minimal deformation occurs during the cutting process thus maintaining high tolerances.

At one end of the flapper, part of the welded hinge is extended 2 inches past the channel width and protrudes through the channel side wall as shown in Fig. 2. A shaft seal serves to minimize any leakage out of the channel. A system of linkages connects the extended portion of the hinge to a servo motor as shown in Fig. 2 (bottom). The linkages are connected to one another with high precision bushing and bearing surfaces, thus providing motion with only one degree of freedom. The servo motor is securely mounted to the top of the channel and is controlled via motion controller from Galil Tools which is connected to a laptop. The motor’s servo provides 4000 counts per revolution which yields a high degree of motion control. The servo system is a closed loop feedback system thus ensuring correct motor shaft position at all times. The linkage system allows the flapper to oscillate with controlled frequency and amplitude.

Steps were taken to reduce natural system noise and verify that the water channel operates “quietly” with no free-stream turbulence. As described previously, water is pumped into the channel via piping through individual pumps. Each pump provides an average flow rate of 30 gpm which passes through 1½ inch pipe and transitions to 4 inch pipe prior to entering the channel. In order to reduce any possible vibrations from the pipes to the channel, the inlet pipes are suspended from above the inlet plenum in such a manner that no physical contact exists between the pipes and the channel. Thus, the only vibrations in the channel are natural system vibrations caused by the flowing water in the channel and the oscillating flapper. The channel was monitored while in operation with the oscillating flapper utilizing an accelerometer. The readings showed that the natural noise of the system was one tenth of that of the oscillating flapper. The streakline shown in Fig. 3 in a $A_t = 0$ flow is evidence that minimal flow disturbances and no free-stream turbulence exists in the channel. The streakline is generated with a dye through a fine needle placed in the flow. The flow rate of the dye is set to match the velocity of the free-stream to avoid disrupting the flow and...
Figure 3: Streakline is shown in the top section of the channel for an $A_t = 0$ flow. The long coherent streakline is evidence that the flow in the channel has no free-stream turbulence. Any free-stream turbulence would quickly mix and dissipate the streak.

thus ensuring accurate flow visualization. If free-stream turbulence were present, this streakline would quickly be mixed and dissipated.

It is imperative that the flow in the top and bottom sections have equal velocity, otherwise the Kelvin-Helmholtz shear instability will exist prior to the bubble and spike formation. With the improved optics, as will be discussed in the next section, a new approach was developed to both measure the velocity more accurately and ensure equal flows of the top and bottom streams. In this new approach dye is injected into the top and bottom streams and photographed at $1 \pm 0.005$ second intervals, and the distance traveled by the dye between successive frames is converted from pixels to distance through a reference photograph of a ruler. Since minimal free-stream turbulence exists, there is no observable deformation on the blob, enabling accurate tracking of its velocity. This procedure is repeated until the velocities are matched using the inlet flow valves. Fig. 4 shows a successive set of photographs with the dye blob in the flow.

Vortex shedding off the flapper was observed if the amplitude for a given frequency was too high, shown in Fig. 5 (top). Therefore, the amplitude is reduced so that the tip velocity is kept one order of magnitude below convective velocity. Multiple rows of bar-grids are placed upstream of the splitter plate to keep the boundary layer small. Although this introduces some small free-stream turbulence, without the grids the boundary layer grows too large and results in leaning of the bubbles and spikes due to the velocity gradients near the centerline, shown in Fig. 5 (bottom).

OPTICAL DENSITY MEASUREMENTS

Following Snider and Andrew [27], density measurements through the span of the test section are obtained optically through the use of the Beer-Lambert’s law. This law relates the spanwise gradient in light intensity $I$ to the molecular absorption $\varepsilon$ and concentration $C$,

$$\frac{\partial I}{\partial z} = -\varepsilon CI. \quad (5)$$

This equation can be integrated a distance, $L$, along the span ($z$ direction) of the water channel from an initial light intensity, $I_0$, to a final light intensity $I$. This yields a relationship between the spanwise-averaged concentration $\overline{C}$ and the drop in light intensity

$$\ln \frac{I}{I_0} = -\varepsilon L \overline{C}, \quad (6)$$

where the mean concentration is defined as

$$\overline{C} = \frac{1}{L} \int_0^L Cdz. \quad (7)$$

as long as: [31, 32]

1. the solute is purely absorptive (no scattering),
2. the solute absorbs light evenly (constant $\varepsilon$),
3. the light is monochromatic,
4. the light is parallel,
5. the optical path from experiment to camera is isolated from other light sources (including reflections).
Figure 5: Top: If the amplitude of the oscillation is too large a nonnegligible vorticity is generated at the edge of the flapper. Bottom: If the boundary layer is too large on the splitter plate, a shearing and leaning of the bubble and spikes is seen.

With respect to a purely absorptive solute, Nigrosine dye is used for its good absorptivity characteristics (negligible scattering). However, if the dye concentration is too high, $\varepsilon$ will not be constant. This constraint is tested by measuring the absorption through a triangular wedge filled with the diluted Nigrosine mixture at a constant concentration. As seen in Fig. 6, the absorption increases linearly with the wedge width, so $\varepsilon$ is constant.

To achieve nearly parallel light, the data acquisition camera is placed a distance of 3 m from the experiment, which also helps reduce parallax uncertainty, as will be discussed later. Light from a pair of fast-charging, repeatable Elinchrom 2x D Lite 400 Ws studio flashes impinges on a diffusion sheet placed behind the channel as shown in Fig. 7. It is this light that passes through the channel and into a Nikon D90 digital camera. To ensure that no other light source is captured by the camera except the light passing through the channel, and to prevent stray light reflections from illuminating the experiment, a black tarp like material is set as a channel between the camera and the channel. Finally, only the red channel data from the digital camera is used in the analysis, effectively reducing the light to a small band of wavelengths (nearly monochromatic). Red is chosen as the Nigrosine slightly scatters blue light (not purely black) and thus also helps further reduce the minimal scattering.

The Schmidt number of the Nigrosine ($\nu/D$) is much larger than the Prandtl number ($\nu/\alpha_{th}$) of the water ($\sim 7$), thus the Nigrosine concentration and temperature are slaved to the fluid motion and not the small but present thermal diffusion [27]. Therefore a measurement of Nigrosine concentration $C$ can be strongly correlated a density $\rho$, with $\rho_2 < \rho < \rho_1$. The non-dimensional span-averaged density as measured in the $n$th image is related to the initial heavy fluid concentration $C_1$

$$f_{1,n} = \frac{\rho - \rho_1}{\rho_2 - \rho_1} \approx \frac{C_n}{C_1} = \frac{-1}{C_1 \varepsilon L} \ln \frac{I_n}{I_0}$$

Figure 6: Absorption of light as a function of width (nondimensional $w/L$) through a triangular wedge filled with the Nigrosine solute. Since the absorption increases linearly with the width the molar absorptivity, $\varepsilon$ is constant.

Figure 7: Top view illustration of camera placement and flashes. The light emanating from the flashes impinges on the diffusion sheet behind the channel, travels through the experiment, and is recorded in the digital camera.
For simplicity of notation, the initial concentration, absorptivity, and channel width will be a single non-dimensional constant $\beta = (C_1 \varepsilon) L^{-1}$. The time and span averaged density $\langle \bar{f} \rangle$ is obtained by averaging over $N$ samples ($N \sim 600$ photographs).

$$\langle \bar{f} \rangle = \frac{1}{N} \sum_{n=1}^{N} f_{1,n} = -\frac{1}{N} \sum_{n=1}^{N} \beta \ln \left( \frac{I_n}{I_0} \right) \tag{9}$$

Although $I_0$ and $\beta = (C_1 \varepsilon) L$ are measurable, to reduce uncertainty the values are set dynamically in each photograph $I_n$ based on the physics. Taking an area average below the splitter plate at the channel entrance where the fluid is purely low-density, the background light intensity is scaled by a factor $I_{0,n} = \eta(I_0)$ such that

$$\frac{1}{A_{bot}} \int \bar{f}_{bot} dA_{bot} = -\frac{1}{A_{bot}} \int \beta_n \ln \left( \frac{I_n}{I_{0,n}} \right) dA_{bot} = 0, \tag{10}$$

and $\langle I_0 \rangle$ is the average over $\sim 100$ images with only water in the channel (not flowing and no nigrosine). In a similar fashion, $\beta$ is set for every image such that an area average above the splitter plate at the channel entrance where the fluid is purely heavy:

$$\frac{1}{A_{top}} \int \bar{f}_{top} dA_{top} = -\frac{1}{A_{top}} \int \beta_n \ln \left( \frac{I_n}{I_{0,n}} \right) dA_{top} = 1. \tag{11}$$

The uncertainty associated with $\beta$ and $I_0$ are the rms of those areas, which is very small with the even backlighting provided by the flashes. For an uneven background lighting, this uncertainty will be large. Following a Kline-McClintock uncertainty analysis, [33] the contributions of $\beta_n$ to uncertainty is:

$$\frac{\partial \langle \bar{f} \rangle}{\partial \beta_n} d\beta_n = \frac{\beta_n}{N} \ln \left( \frac{I_n}{I_{0,n}} \right) \tag{12}$$

Similarly, the uncertainty from the background is

$$\frac{\partial \langle \bar{f} \rangle}{\partial I_{0,n}} dI_{0,n} = \frac{\beta_n - I_n / I_{0,n}}{N} dI_{0,n} = \frac{\beta_n I_{0,n,rms}}{N} \tag{13}$$

The uncertainty associated with the $n$th image is:

$$\frac{\partial \langle \bar{f} \rangle}{\partial I_n} dI_n = -\frac{\beta_n (dI_n / I_{0,n})}{N} dI_n = \frac{\beta_n dI_n}{N I_n} \tag{14}$$

Over multiple photographs, any variation (rms) of this static (no flow) set of photographs is thus associated with optical uncertainty

$$\frac{dI_n}{I_n} = \frac{I_{w,rms}}{I_w}, \tag{15}$$

where $I_w$ represents the wedge images. Just like the background images, this is averaged over $\sim 100$ images. Therefore, a repeatable backlighting and image acquisition system will generate very small uncertainty.

The overall uncertainty in mixing fraction $d\langle \bar{f} \rangle$ is then:

$$d\langle \bar{f} \rangle^2 = \sum_{n=1}^{N} \left( \frac{\beta_n}{N} \ln \left( \frac{I_n}{I_{0,n}} \right) \right)^2 + \left( \frac{\beta_n (I_{0,n,rms})}{N I_{0,n}} \right)^2 \tag{16}$$

With the studio flashes and high end camera, the differences between each successive $\beta_n$ and $I_{0,n}$ are very small (less than 0.1% variation per image), the above uncertainty equation reduces to

$$d\langle \bar{f} \rangle^2 \approx (\beta)^2 \left[ \left( \frac{\langle \beta_{rms} \rangle}{\langle \beta \rangle} \ln \left( \frac{I_n}{I_0} \right) \right)^2 + \left( \frac{\langle I_{0,rms} \rangle}{I_0} \right)^2 + \left( \frac{I_{w,rms}}{I_w} \right)^2 \right]. \tag{17}$$

Data acquisition was performed with a Nikon D90 digital camera. To ensure no additional optical errors, circle of confusion ($c$) of the camera is set to one pixel in order to calculate the appropriate camera aperture. For the Nikon D90 with a sensor size of 23.6 mm, $c = 5.5 \mu m$, for a depth of field (DOF) that includes the entire channel. The DOF of the image is based on the camera lens focal length $F$ and aperture size $F/M$.

$$\text{DOF} = ZF^2 \left( \frac{1}{F^2 - M c (Z - F)} - \frac{1}{F^2 + M c (Z - F)} \right). \tag{18}$$

With a focal length of $F = 105$ mm and aperture setting of $M = 16$, the DOF is 371 mm (14.6 inches) which more than covers the entire width of the channel thus ensuring the test section is in focus. With the resolution of $4288 \times 2848$, the one pixel spans 0.25 mm of the experiment. At a convective velocity of $U=5.7 \text{ cm/s}$, the exposure time is set to 1/320 s with studio flash duration of 1/1200 s to ensure no blurring. This combination of small aperture and quick exposure time required the installation of studio flashes to provide the adequate backlight intensity to
maintain the high quality image (ISO 200) on the Nikon D90. Images are captured at a 1 Hz rate, resulting in ~ 600 images per test case.

**UNCERTAINTY ANALYSIS**

The experiment as described in the previous section is designed to obtain a mixing height $h$ versus time $t$ for different initial condition wavelengths $\lambda$ by measuring the distance between $\langle \bar{I} \rangle = 0.95$ and 0.05 (eqn. 2). As described previously, the goal of this work is to characterize the mixing height $h$ on time and initial wavelength, but it is also dependent on Atwood number, gravity, viscosity, and system height $H$:

$$h = h(t, \lambda; A_t, g, \nu, H). \tag{19}$$

To report the data non-dimensionally for each case and for apparatus independency, appropriate parameters for non-dimensionalizing $h$, $\lambda$, and $t$ need to be chosen. For $h$, the natural choice is to non-dimensionalize with the system height $H$. However, effort is taken to make a system height large enough so that it does not affect $h$. Therefore $H$ is inappropriate choice for non-dimensionalization, as it would be apparatus dependent and unrelated to the physics. Another standard choice, especially when examining late time, is to non-dimensionalize with reduced gravity $Ag$ and time $(Ag)^2$, thus effectively fitting the data to Young’s model (also known as “alpha”) [7]. For early time, however, this choice is also not appropriate. Additionally, this choice does not keep the non-dimensional $h$ decoupled from $t$ for characterization purposes. The only choice remaining is viscosity, which does play a minimal role in the development of the most unstable wavelength, growth of the secondary flows, and the dissipation of energy. Therefore, time and height are non-dimensionalized with the reduced gravity $A_t$, and viscosity $\nu$:

$$\mathcal{H} = h(\nu^2/(A_t g))^{1/3}, \tag{20}$$

$$\tau = t((A_t g)^2/\nu)^{1/3}, \tag{21}$$

$$\lambda^* = \lambda(\nu^2/(A_t g))^{1/3} \tag{22}$$

such that the dependency is now $\mathcal{H} = \mathcal{H}(\tau, \lambda^*)$. This is akin to non-dimensionalizing $h$ with the most unstable wavelength (eqn. 4), but without the extra $4\pi$ factor. With this nondimensionalization, the channel height is $\sim 395$. The uncertainty $d\mathcal{H}$ will be dependent on mixing height uncertainty $dh$, Atwood uncertainty $dA_t$, viscosity uncertainty $d\nu$, and gravity uncertainty $dg$,

$$\frac{d\mathcal{H}}{\mathcal{H}} = \sqrt{\left(\frac{dh}{h}\right)^2 + \left(\frac{2}{3} \frac{d\nu}{\nu}\right)^2 + \left(\frac{1}{3} \frac{dg}{g}\right)^2 + \left(\frac{1}{3} \frac{dA_t}{A_t}\right)^2}. \tag{23}$$

The uncertainty of the mixing height $dh$ has two sources, an uncertainty in $y$ from the parallax of the camera ($dy$), and an uncertainty due to resolving the location of $\langle \bar{I} \rangle = 0.95$:

$$\frac{dh}{h} = \sqrt{\left(\frac{dy}{h}\right)^2 + \left(\frac{d\langle \bar{I} \rangle}{h} \left(\frac{\partial \langle \bar{I} \rangle}{dy} \bigg|_{\langle \bar{I} \rangle = 0.95}\right)^{-1}\right)^2}, \tag{24}$$

For this analysis, low-Atwood symmetry is applied such that $h = y_{\langle \bar{I} \rangle = 0.95}$.

Camera parallax, light not normal to the camera plane, has two affects in the optical measurements, depicted in Fig. 8. The first is the additional distance the light traverses within the experiment results in a further decrease in light intensity [27]. This intensity decrease is accounted for by proportionally scaling the measured light intensity [27]. The second effect is the difference coordinate location of the entering and exiting light ray. For example, light that is incident on the camera but originates far downstream may enter the channel at 60 cm and exit the channel at 58 cm. The location is taken to be the average of the entrance and exit point, and the difference between the two is incorporated into the overall error analysis as position uncertainty $dx$ and $dy$:

$$dx = \frac{w(v_{water})}{v_{air}} \sqrt{x^2 + \left(1 - \left(\frac{v_{water}}{v_{air}}\right)^2\right)}, \tag{25}$$

$$dy = \frac{w(v_{water})}{v_{air}} \sqrt{y^2 + \left(1 - \left(\frac{v_{water}}{v_{air}}\right)^2\right)}, \tag{26}$$

where $v_{water}$ and air $v_{air}$ is the speed of light in water and air, respectively. The length the light travels in the channel is $L = \frac{w(v_{water})}{v_{air}} \sqrt{\langle \bar{I} \rangle - 0.95}$. 

![Figure 8: Parallax effects: light travels along distance $L$, not channel width $w$, and is accounted for by proportionally scaling the measured intensity. The $x$ and $y$ coordinate of the light is taken as the average between the entrance and exit location, with the difference of the two included as position error in the uncertainty analysis.](image-url)
\[ \sqrt{dx^2 + dy^2 + w^2}. \] The distance from the camera to the channel was 20 feet, which is always much greater than the distance \( x \) or \( y \), resulting in a relatively constant \( dx/x \sim dy/y \sim 2.4\% \).

The Atwood uncertainty is

\[ \frac{dA}{A} = \frac{1}{\rho_1 - \rho_2} \sqrt{((1-A)d\rho_1)^2 + ((1+A)d\rho_2)^2}. \] (27)

This error was reduced by using a densitometer from Rudolph, Inc. with a resolution of \( d\rho_1 = d\rho_2 = 0.00005 \) g/cm\(^3\). This brought the initial density uncertainties of \( d\rho_1 \) and \( d\rho_2 \) to approximately 0.005\% for an Atwood uncertainty of \( dA/A \sim 4\% \). This is lower than using the densities based off NIST curves for water and also accounts for any seasonal variation in College Station municipal water.

Gravity is \( g = 9.793 \) m/s\(^2 \) (local to College Station, TX), taken with a 0.5\% uncertainty. The viscosity of water taken from NIST data at the average temperature between hot and cold streams \( \pm 1^\circ C \) (\( \nu = 7.6 \pm 0.14 \times 10^{-7} \) m\(^2\)/s).

With the measurement of convective velocity discussed in the experimental setup section, velocity and time uncertainties are \( dU/U = \sqrt{(dx/x)^2 + (dt/t)^2} \sim 2.45\% \), \( dt/t = \sqrt{dx/x}^2 + (dU/U)^2 \sim 3.43\% \), such that

\[ \frac{d\tau}{\tau} = \sqrt{\left(\frac{dt}{T}\right)^2 + \left(\frac{dv}{3\nu}\right)^2 + \left(\frac{2\,dg}{3\,g}\right)^2 + \left(\frac{2\,dA}{3\,A}\right)^2} \sim 3.53\%. \] (28)

With the Atwood uncertainty reduced to 4\%, along with the 1/3 factor in eqn. 24, the largest and most dominant component of the mixing height uncertainties is the parallax at 2.4\%. Thus, the best possible measurements will occur if the uncertainty propagated to \( dh \) through the density optical measurements are under this amount.

**RESULTS**

The effect of single mode initial conditions on the mixing height growth rate \( h \) are examined using optical density measurements in the low-Atwood water channel facility at Texas A&M, as described in the previous sections. Sample pictures of the experiment for each wavelength case are shown in Fig. 9, and Table 1 describes the single mode experiments reported in this paper. The table is split into five columns with giving the initial amplitude \( h_0 \), period \( T \), wavelength \( \lambda \), non-dimensional wavelength \( \lambda^* \), Froude number relating the maximum vertical speed of the imposed initial condition compared to saturation growth rates \( (Fr = 2\pi h_0/(T\sqrt{A\,g\,\lambda}) \), and Atwood number for each run. In the following, Run 1 is considered the baseline case and com-
Figure 10: Average density $\langle T_1 \rangle$ for the baseline, 2, 3, 4, 6, and 8 cm cases (top to bottom, left to right) as a function of height above the flapper and downstream distance (cm).

Figure 11: Average density uncertainty $d\langle T_1 \rangle$ for the baseline, 2, 3, 4, 6, and 8 cm cases (top to bottom, left to right) cases as a function of height above the flapper and downstream distance (cm). After the optical modifications, the relative error is now at most 1%.
Table 1: Values of the flapper amplitude $h_0$ and period $T$, resulting initial condition wavelength $\lambda$, normalized initial condition wavelength $\lambda^* = \lambda(A_{ig}/\nu^2)^{1/3}$, convective velocity $U$, Froude number $(2\pi h_0/(T \sqrt{A_{ig}g}))$ and Atwood number $A_t$, from densitometer measurement.

<table>
<thead>
<tr>
<th>Run</th>
<th>$h_0$ (mm)</th>
<th>$\lambda$ (cm)</th>
<th>$\lambda^*$</th>
<th>$U (\frac{cm}{T})$</th>
<th>Fr</th>
<th>$A_t \times 10^{-4}$</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.71</td>
<td>-</td>
<td>9.64</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>2</td>
<td>46.2</td>
<td>5.71</td>
<td>0.080</td>
<td>9.29</td>
</tr>
<tr>
<td>3</td>
<td>0.31</td>
<td>3</td>
<td>69.2</td>
<td>5.71</td>
<td>0.072</td>
<td>9.29</td>
</tr>
<tr>
<td>4</td>
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<td>92.5</td>
<td>5.71</td>
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<td>6</td>
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<td>5.71</td>
<td>0.14</td>
<td>12.0</td>
</tr>
<tr>
<td>6</td>
<td>0.82</td>
<td>8</td>
<td>196</td>
<td>5.71</td>
<td>0.12</td>
<td>12.0</td>
</tr>
</tbody>
</table>

prises of no oscillations where the only disturbances to the interface are the natural disturbances associated with the experiment. The lowest applied wavelength of 2 cm was chosen based on the baseline most unstable wavelength calculation $\lambda_\text{m}$, as shown in equation 4, which for this experiment was approximately 5 mm.

The applied wavelength in each case grows to be the dominant wavelength, although it is not as evident in the 2 and 3 cm cases since the wavelengths are closer to the natural wavelengths already present in RT mixing. The boundary layer off the flapper is approximately 5 mm per side, resulting in a 1 cm wake off the flapper tip that can be visually seen in the larger wavelength cases. The classic Rayleigh-Taylor bubbles and spikes are evident, and increasing in size for the larger initial conditions. No leaning or complex interactions from the initial conditions were observed.

The average density $\langle \rho \rangle$ and uncertainty $d\langle \rho \rangle$ contours for all cases are similar, shown in Figs. 10 and 11, respectively. With the improvements made to the experiment, the density uncertainties are at most 1%. With this improved accuracy, the subtle differences due to the initial conditions are now quantifiable.

The average mixing height $\langle H \rangle$ as shown in Fig. 12 (top) grows linearly for the large initial conditions case, and slightly quadratically for the baseline case. A quadratic curve fit for the baseline case from $5 \leq \tau \leq 20$ is $\langle H \rangle = 5.6 + 2.7\tau + 0.037\tau^2$ with $R^2 = 0.996$, showing a very weak quadratic dependence. The 95% confidence interval on the quadratic term is between 0.033 and 0.043. Likewise, using a power law fit yields $\langle H \rangle = 1.6\tau^{1.24} + 8.4$ with $R^2 = 0.996$ and a 95% confidence interval on the exponent between 1.20 and 1.27. The 2, 3, and 4 cm cases, within uncertainty (Fig. 12 bottom), all seem to start with a terminal velocity, and then leave the common terminal velocity line and approach the slightly quadratic baseline growth rate. A larger facility would be required to determine if the 6 and 8 cm cases would ultimately follow suit and approach the baseline case.

The uncertainty $d\langle H \rangle = dh(A_{ig}/\nu^2)^{1/3}$ is shown in Fig. 12 (bottom) and includes by its normalization the uncertainty in Atwood and viscosity. Relative error starts near 15% but tapers down to 5% near $\tau = 15$. This is due to the problem of resolving

![Figure 12: Mixing height $\langle H \rangle = h(A_{ig}/\nu^2)^{1/3}$ versus time $\tau = t(A_{ig}/\nu)^{1/3}$ for the baseline, 2, 3, 4, 6, and 8 cm perturbations. The no perturbation case has a slight quadratic dependency. Each of the initial conditions starts on a terminal velocity (straight line) and eventually deviate towards the no perturbation case. The water channel facility is not large enough to determine when or if the 6 and 8 cm perturbations would follow the same trend. All cases show the same growth rate within uncertainty, $\epsilon = d\langle H \rangle/\langle H \rangle$ which starts near 15% but tapers down to 5% near $\tau = 15$ as more pixels span the mixing width.](image-url)
the location of $\langle f_1 \rangle = 0.95$ as discussed in the uncertainty section. At early time the mixing width is spanned by a small set of discrete pixels, and as the mixing width grows there are more pixels to more accurately resolve the location of $\langle f_1 \rangle = 0.95$. However, at later times the uncertainty asymptotes to $\sim 5\%$. Nevertheless, at those late times ($\tau \sim 30$) the experimental uncertainty is slightly less than the difference between the $\mathcal{H}$ curves and so the effect of initial condition is a quantifiable difference.

In observing the growth rates of the height between $\langle f_1 \rangle = 0.10$ and $0.90$, and $0.20$ and $0.80$ (instead of the standard $0.05$ and $0.95$), the rate at which the mix is homogenized within the mixing layer is observed. As shown in Fig. 13 for $0.90/0.10$ (top: $\mathcal{H}_{0.9}$) the larger wavelength initial conditions tend to result in a more homogenized mix. This separation between the initial conditions is more significant for $0.80/0.20$ (bottom: $\mathcal{H}_{0.8}$). Qualitatively, this is observed in the average density plots in Fig. 10 as the $8$ cm initial condition case shows a much larger yellow region marking the fully mixed case $\langle f_1 \rangle = 0.5$.

**CONCLUDING REMARKS**

The reduction of the optical density measurement uncertainty to $1\%$, below the reduced Atwood uncertainty ($4\%$) and parallax ($2.4\%$), enabled the uncertainty in mixing height $d\mathcal{H}/\mathcal{H}$ to asymptote to $5\%$. Along with the reduced $d\tau/\tau \sim 3.5\%$, this low uncertainty allowed a quantification of the effect of single wavelength initial conditions on the mixing width growth rate in the low-Atwood regime. The observed differences in the initial conditions are outside the experimental uncertainty, thus proving the hypothesis that initial condition effects on mixing layer growth exist in the low-Atwood regime.

The observed differences in single-mode initial conditions are that the $2$ cm and $3$ cm wavelengths approach to the baseline condition. The $4$ cm wavelength seems to start to approach the baseline condition as well, however, the channel’s physical domain is too small to determine if that is indeed the case. Similarly, the $6$ cm and $8$ cm wavelength may at later time approach the baseline condition. The evaluation of much later-time growth rates is of interest and should be explored to determine whether full asymptotic relationship to the baseline case exists. Within the mixing layer, the larger wavelength initial conditions homogenize the mix faster.

The current research is primarily interested in early time evolution of RT mixing. The effects of initial conditions are observable at low-Atwood. With the hypothesis proven that the effect of initial conditions on mixing growth rates exist in the low-Atwood water channel facility, the effects of multi-mode and complex initial conditions are now accessible. Different combinations of initial conditions can now be explored, leading to the possibility of modifying and thus designing turbulent mixing growth rates.

**ACKNOWLEDGEMENTS**

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NOMENCLATURE

Upper-case

H  Nondimensional height
A  Area (m^2)
A_t  Atwood number \((\rho_1 - \rho_2) / (\rho_1 + \rho_2)\)
C  Concentration (mol/m^3)
D  Diffusivity of Nigrosine (m^2/s)
F  Focal length (m)
H  Total system height (m)
I  Light intensity (varies spatially) (W/m^2)
L  Distance light travels inside the channel (m)
M  Aperture ratio
N  Total number of images
P  Pressure (Pa)
R  Least-squares residual (curve-fitting)
U  Mean streamwise velocity (m/s)
Z  Distance from channel to camera (m)
Fr  Froude number
Re  Reynolds number
T  Period of oscillation (s)

Lower-case

c  Circle of confusion (m)
d  Uncertainty of (e.g. \(dh\), uncertainty of \(h\))
f  Density (non-dimensional)
g  Gravity (m/s^2)
h  Mixing height (m)
t  time w.r.t. mixing \((x/U)\) (s)
v  Speed of light in media (m/s)
w  Width of channel (m)
x  Streamwise direction (m)
y  Normal direction (direction of mixing) (m)
z  Spanwise direction (direction of light) (m)

Greek

\alpha  Growth rate parameter
\alpha_{th}  Thermal diffusivity (m^2/s)
\beta  \((\bar{C}^2\epsilon L)^{-1}\)
\epsilon  Molar Absorptivity of light (m^2/mol)
\eta  Background intensity \((\eta_0, n = \eta_0)\) scaling factor
\lambda  Wavelength (m)
\lambda^*  Nondimensional initial wavelength
\lambda_{mn}  Most unstable wavelength (m)
\nu  Kinematic viscosity (m^2/s)
\rho  Density (kg/m^3)
\tau  Nondimensional time

Subscripts

b  Bubble
m  mth image
n  n-th image
rms  Root-mean-square
s  Spike
w  Wedge
0  Initial

1  Heavy fluid
2  Light fluid
bot  Area below splitter plate (used to set \(\eta\))
top  Area above splitter plate (used to set \(\beta\))

Symbols

⟨⟩  Sample-averaged
—  Spanwise-averaged

Acronyms

DOF  Depth of Field (m)
DT  Deuterium-Tritium
ICF  Inertial Confinement Fusion
NIST  National Institute for Standards and Technology
RT  Rayleigh-Taylor

REFERENCES


