Low Pressure Turbine Relaminarization Bubble Characterization using Massively-Parallel Large Eddy Simulations

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The separation and reattachment of suction surface boundary layer in a low pressure turbine is characterized using large-eddy simulation at $Re_{s*} = 69,000$ based on inlet velocity and suction surface length. Favorable comparisons are drawn with experiments using a high pass filtered Smagorinsky model for sub-grid scales. The onset of time mean separation is at $s/s_o = 0.61$ and reattachment at $s/s_o = 0.81$, extending over 20% of the suction surface. The boundary layer is convectively unstable with a maximum reverse flow velocity of about 13% of freestream. The breakdown to turbulence occurs over a very short distance of suction surface and is followed by reattachment. Turbulence near the bubble is further characterized using anisotropy invariant mapping and time orthogonal decomposition diagnostics. Particularly the vortex shedding and shear layer flapping phenomena are addressed. On the suction side, dominant hairpin structures near the transitional and turbulent flow regime are observed. The hairpin vortices are carried by the freestream even downstream of the trailing edge of the blade with a possibility of reaching the next stage. Longitudinal streaks that evolve from the breakdown of hairpin vortices formed near the leading edge are observed on the pressure surface.

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1 Introduction

Many flows of engineering and academic interest have adverse pressure gradients and hence are susceptible to separation. If separated, the point of separation and reattachment (if any) are often very difficult to predict. Furthermore, the separation point is often geometry and case dependent. In turbo-machinery, specifically in low pressure turbines (LPT), both separation points are further complicated by boundary layer relaminarization as the free-stream is accelerated around the blade, and at certain characteristic Reynolds numbers, transition back to turbulence and then reattachment [3,8].

This relaminarization, separation, transition, and reattachment is known as the relaminarization bubble, and the full details of the flow physics involved are very difficult to obtain experimentally. Therefore, highly accurate, massively-parallel time-dependent simulations can provide insight into the relaminarization bubble. Due to the unsteady nature of the flow and the relatively large amount of data created, effective analytical tools are also required to mine the data to extract the relevant physics and phenomena.

In this paper we present results of a large eddy simulation (LES) investigation of the relaminarization bubble found in a low pressure turbine at a Reynolds number $Re_{s*} = 69,000$ based on freestream velocity and suction surface length. The resulting 3-D time dependent turbulent flow field is analyzed prior, during, and after reattachment.

2 Background

Due to the high accelerations around the LPT, the boundary layer relaminarizes marking the starting point of the laminar “bubble.” Investigations in the past have focused on several areas in separation bubble dynamics: classification of bubbles, its dependence on FSTI and Re, prediction of onset and reattachment point, effect of flow upstream on bubble and the type of transition (natural and bypass). Gaster [9] pioneered the early work on separation bubbles and identified the bursting mechanism with separation based Re and pressure distribution in the bubble region. Gaster also mentions about a low frequency motion of the bubble that is more pronounced and violent in long bubbles. This forms one of the motivation for the present study. Horton [10] then deduced conditions for reattachment of the shear layer and arrived at correlations for nondimensional bubble length of laminar and turbulent regions separately. Volino [11] investigated the movement of the separation bubble, onset and reattachment and its dependence on Reynolds number and found that at high Reynolds number, separation and attachment occurred in quick succession independent of FSTI, however higher FSTI resulted in higher loss. But in an actual gas turbine, the separation bubble dynamics is further complicated by blade-row interactions that results in unsteady wakes. Transition induced due to the unsteady wakes are known as wake-induced transition. Wake-induced transition and its associated losses summarized by Hodson [12] are not discussed here, since it forms part of our future study. Influence of wake structures have been investigated by [13–17] among many others and the reader is also referred to the references therein.

The transition of the boundary layer from laminar back to turbulence begins with a Kelvin-Helmholtz instability of the shear layer, the growth of which results in formation of spanwise roll-up vortices. The subsequent breakdown of these spanwise vortices into smaller structures is believed to cause the transition to turbulence and thus the reattachment [3]. Two types of instability could be associated with the growth of disturbance namely, convective and absolute instability. Spatial propagation of instability is convective in nature while propagation of disturbance with time leads to absolute instability [2]. Alam and Sandham [1] deduced

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a criterion for classifying the instabilities based on the reverse flow velocity inside the bubble. The threshold remains in the range of 15–20% of freestream velocity for convective instabilities.

Transition to turbulence occurs after instability growth. Yang and Voke [2] observed hairpin vortices during transition and vortex shedding from a separated shear layer in their LES study. The shedding process has been observed to be aperiodic with a single frequency and a range of frequencies has been established. A flat plate fails to replicate the exact flow physics of an LPT cascade, such as the distortion of wakes at the leading edge [18]. It is therefore essential that the transition mechanism be quantified under the conditions present in turbomachinery applications as it is of direct practical interest to engineers.

At current computational resources, a direct numerical simulation (DNS), which resolves all length scales of turbulence, is too expensive. Therefore, Large Eddy simulation is used. However, when performing LES, a sub-grid scale (SGS) model is essential to provide sufficient dissipation in underresolved regions of flow. LES investigations by Wilson et al. [19] did not yield an agreeable result with the experiments of Gaster [9] when simulated without SGS modeling. A key aspect of reproducing the correct physics computationally involves using a turbulence model that performs well in regions of high acceleration and separation, where standard LES (and RANS) models perform poorly.

Table 1 summarizes geometry types and resolution in wall units (xw = uτ/ν) nondimensionalized with shear velocity uw = √τw/ν (based on wall shear τw and density ρ) and viscosity ν. As can be seen, many simulations have been done with a flat plate geometry and inducing a pressure gradient by contouring the wall, suction or by other means to replicate the conditions that would trigger transition. Michelassi et al. [7] attempted to characterize the flow structures on the suction and pressure surface with the presence of wakes. Their study yielded agreeable results for the pressure surface but the fine scale structures on the suction side were not captured because of under-resolved regions. The simulation by Kalitzin et al. [4] suffered significantly from low grid resolution near the leading and trailing edge of the blade and skewness in the passage showing signs that an accurate and highly resolved DNS with the current resources is far-fetched. A highly resolved LES capturing the dominant flow structures to quantify the transition mechanism and kinematics of the separation bubble within an LPT cascade is still missing.

Using Large Eddy simulation, we investigate the transition mechanism and flow structures on the suction and pressure surface as well as the shear layer flapping phenomenon using time orthogonal decomposition (TOD) diagnostics on the separation bubble. Sarkar [13] previously investigated the flow structures on a low pressure turbine using proper orthogonal decomposition (POD) in the presence of upstream wakes illustrating the dominant energy containing modes. However the shear layer flapping was not addressed.

### Table 1 \ Review of grid resolutions used and type of study conducted for previous separation studies. Computational investigations based on LPT suffered from low grid resolution, failing to capture the finer scales of motion

<table>
<thead>
<tr>
<th>Authors</th>
<th>Method</th>
<th>Δxw</th>
<th>Δyw</th>
<th>Δzw</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alam and Sandham</td>
<td>DNS</td>
<td>14–20</td>
<td>0.5</td>
<td>~1.0</td>
<td>Flat plate</td>
</tr>
<tr>
<td>Yang and Voke</td>
<td>LES</td>
<td>10–30</td>
<td>1</td>
<td>9</td>
<td>Flat plate</td>
</tr>
<tr>
<td>McAuliffe et al.</td>
<td>LES</td>
<td>19</td>
<td>0.5</td>
<td>19</td>
<td>Flat plate</td>
</tr>
<tr>
<td>Kalitzin et al.</td>
<td>DNS</td>
<td>28</td>
<td>2.3</td>
<td>1.9</td>
<td>Turbine</td>
</tr>
<tr>
<td>Roberts and Yaras</td>
<td>LES</td>
<td>36</td>
<td>0.8</td>
<td>38</td>
<td>Flat plate</td>
</tr>
<tr>
<td>Wissink et al.</td>
<td>LES</td>
<td>40–60</td>
<td>2–4</td>
<td>10–20</td>
<td>Turbine</td>
</tr>
<tr>
<td>Michelassi et al.</td>
<td>LES</td>
<td>65</td>
<td>&lt;5.5</td>
<td>15–20</td>
<td>Turbine</td>
</tr>
</tbody>
</table>

### 3 Numerical Method
A geometrically flexible spectral element solver, NEK5000 [20], is employed that solves for conservation of mass and momentum by decomposing the domain into K nonoverlapping elements and approximating the solution in each element as series of high order polynomials. Along with the orthogonal polynomial basis functions and gaus-siobatto-legendre (GLL) quadrature in the reference domain, a high parallel efficiency is achieved [21]. The attractive feature of Spectral element method is the affordability of exponential convergence [22]. The conservation of mass and momentum (Navier-Stokes equations) for an incompressible fluid are:

\[
\frac{\partial U_i}{\partial t} + \nabla \cdot (U_i U_j) = 0
\]  

(1)

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} + \frac{1}{2} \tau_{ij}^{SGS} \right)
\]  

(2)

with velocity vector U_i, density ρ, pressure P, kinematic viscosity υ, and sub-grid stress τ_{ij}^{SGS} (LES turbulence model). The solver has shown strong scaling up to 1024 processors for the present study, shown in Fig. 1.

#### 3.1 High Pass Filtered Smagorinsky Model
Computational simulations using steady state solutions of the Navier-Stokes equations (Reynolds-Averaged Navier-Stokes (RANS)) and turbulence models based on the Boussinesq approximation:

- \[ -\pi \tau_{ij} = 2\nu_i S_{ij} + \frac{1}{2} k \delta_{ij} \]  

(3)

- \[ \bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \]  

(4)

- \[ u_i = U_i - \bar{U}_i \]  

(5)

often fail to predict transition completely due to the high anisotropy associated with the flow. Here, \( \bar{S}_{ij} \) is the mean rate of strain given by Eq. (4), \( \nu_i \) is the eddy viscosity, \( k \) is the turbulent kinetic energy defined as \( k = u' \bar{u}' / 2 \), \( \delta_{ij} \) is the Kronecker δ and \( \pi \tau_{ij} \) is the Reynolds stress tensor. Furthermore, RANS models produce large strain in the stagnation region and tend to over-predict the

![Fig. 1 Strong scaling of NEK5000 for two different computer architectures: (square) EOS at Texas A&M Supercomputing Center and (circle) Ranger at Texas Advanced Computing Center(TACC). When the number of processor is doubled, the computational time per step should decrease by one-half for ideal scaling (dashed lines).](image-url)
turbulent kinetic energy that affects the flow downstream of the blade [4].

Therefore, unsteady solutions with sub-grid stress models (LES) are needed. Typically in classical LES modeling (Smagorinsky model), the transition to turbulence is not predicted since the subgrid stress is over-estimated (dissipative), which can be overcome by using a dynamic version of the model [23]. Another model that appears to have the ability to predict transition is a high-pass filtered (HPF) Smagorinsky model developed by Stolz et al. [24]. Since the model is based on HPF velocity strain-rate, the eddy viscosity vanishes in laminar flow regimes which is essential in transitional flows. Secondly, a reduced eddy viscosity is predicted near the walls which eliminates the need to use near-wall damping functions. The model employs high-pass filtered quantities for calculating the eddy viscosity and strain rate:

\[
\rho^{-1} \left( \frac{\varepsilon_{SGS}}{\Delta} - \frac{\delta_{ij}}{3} \frac{\varepsilon_{SGS}}{\Delta} \right) = -2 \nu_{i}^{HPF} S_{ij}(\tilde{u}) \tag{6}
\]

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_{i}}{\partial x_{j}} + \frac{\partial \tilde{u}_{j}}{\partial x_{i}} \right) \tag{7}
\]

\[
\nu_{i}^{HPF} = \left( \frac{C_{t}^{HPF}}{C_{0}} \right)^{2} |S_{ij}(\tilde{u})| \tag{8}
\]

where \( \Delta \) is the filter width taken as the largest distance between the gauss-lobatto-legendre (GLL) points in physical space. \( C_{i}^{HPF} \) is a constant with a value 0.1, and \( \tilde{u} \) is the high pass filtered velocity field. A dynamic version of the model with varying \( C_{t}^{HPF} \) is possible; however, the fixed co-efficient has shown satisfactory results in Stolz [24] and is thus chosen for the present investigation.

### 3.2 Validation of the HPF Model

The HPF Smagorinsky model was tested and appropriate high-pass-filter transfer function was found by setting up a turbulent flow over a flat plate with a hemispherical protuberance. The results are validated against a DNS at the same Reynolds number. The flow domain is shown in Fig. 2. A recycling and rescaling method introduced by Stolz et al. [25] is used to generate turbulence at the inlet. The mean velocities are scaled for constant volume averaged velocity inside the recycling box and copied back to the inflow plane. The inflow and recycling planes are located at \( Z/D = -5.0 \) and \( Z/D = -0.22 \) (\( D \) is the diameter of the hemisphere), respectively. The stream function of the velocity field at the inlet is given by Ref. [26]:

\[
\psi = \frac{1}{4} \cos(3\zeta) \sin(4\eta) - \frac{1}{5} \cos(5\zeta) - \frac{1}{5} \sin(5\zeta) \tag{9}
\]

\[
-\nabla^{2} \psi = \frac{1}{\psi} \tag{10}
\]

where \( \psi \) is the streamfunction and \( \zeta \) is the Eigenvalue of the Laplacian. This allows parametric control of freestream turbulence intensity and length scales by varying the amplitude and wavenumbers of the sine and cosine functions, respectively. Increasing the wavenumbers results in smaller vortices at the inflow plane. A simple proportional-integral-derivative (PID) controller is used to control the turbulent intensity (TI) inside the recycling domain. At each time step, the spatial intensity in the recycling box is compared to a desired value. If too low, additional vortices are added by combining a percentage of Eq. (9) with the copied recycling plane flow. If the intensity is too high, no vortices are added and the viscosity naturally dampens the fluctuations.

At \( Z/D = 0 \), a flat plate initiates the formation of a boundary layer upstream of the hemisphere that is located at \( Z/D = 2.0 \). A stress free boundary condition is used in the pitchwise direction for the nonrecycling domain. The streamwise velocity and rms profiles at different downstream locations of the hemisphere are considered for comparison against the DNS. The two different filter functions tested are shown in Fig. 3. Additionally, a stabilizing filter developed by Fischer et al. [27] is used for the spectral element basis function that filters out the high frequency components shown in Fig. 3. The filter is applied after each time step and also preserves inter-element continuity [22]. This avoids the non-physical spectral build up in the energy spectra that is usually found in under-resolved DNS or LES. The HPF Smagorinsky model is applied after each time step and acts upon the velocity components filtered by the stabilizing filter. A convergence study for the DNS is reported in Fig. 4. This shows exponential convergence typical of spectral methods, and that a polynomial order 14 is sufficient for the DNS.

Time-averaged mid-section velocity profiles at 0.5D, 1.5D, 2.5D downstream of the hemisphere are shown in Fig. 5. The velocity profiles are in reasonable agreement except at 2.5D where Filter 1 performs better near the boundary layer and is thus chosen for production runs. The corresponding rms velocity profile for Filter 1 is shown in Fig. 6. While there are a few discrepancies between the LES and DNS rms profiles, the LES follows the trend predicted by the DNS in most flow regimes.

### 3.3 LPT Computational Domain

A single blade cascade with periodic boundary conditions in the pitch and span-wise direction is considered. The Reynolds number, cascade angle, solidity ratio and turbulence intensity are matched to the experimental set up of Schoberi et al. [8]. The computational grid extends 0.725 \( C_{as} \) downstream of the trailing edge of the blade and about 5 \( C_{as} \), upstream of the cascade including the turbulence generating section. Only about 35% of the blade height (~0.4\( C_{as} \)) is simulated. The convergence plot for the LPT simulation is shown in Fig. 7. All the plots thus shown are converged solutions. A nearly isotropic turbulence is generated by allowing a divergence free velocity field with multiple vortices at the inflow plane, \( X/C_{as} = -5.0 \), to advect inside a doubly periodic box. The stream function of the velocity field is given by Eq. (9), where the wave numbers of the sines and cosines follow the relation \( \lambda^{2} = m^{2} + n^{2} \), where \( m,n \) belong to the class of Natural numbers. At \( X/C_{as} = -4.50 \), an array of square bars redistributes the turbulence kinetic energy and turbulence is allowed to decay. The computational domain without the periodic boundaries is shown in Fig. 8.

The periodic box does not introduce any shear layer and thus preserves the vortical structures in the flow. The root mean square (rms) of streamwise, pitchwise and spanwise velocity components show a similar trend in magnitude and decay rate as shown in Fig. 9, which is characteristic of isotropic turbulence. The flow is simulated under...
Fig. 3 The filter transfer functions are set similar to a cosine function that filters the large scales and leaves the small scales unfiltered. A stabilizing filter is used for all simulations (DNS and LES) where the last 4 modes are filtered with a weight of 0.05.

Fig. 4 A fairly constant slope on a semilog plot of error versus polynomial order signifies an exponential convergence for the hemisphere DNS. Data is acquired at a polynomial order of 14.

Fig. 5 Streamwise velocity profiles at 0.5D, 1.5D, 2.5D downstream of the hemisphere, offset by 0.5, respectively. The profiles are in good agreement with the DNS, except at 2.5D where Filter1 performs better predicting the boundary layer growth in accordance with the DNS.

Fig. 6 Streamwise fluctuation velocity profiles at 0.5D, 1.5D, 2.5D downstream of the hemisphere, offset by 0.2, respectively. The filtered rms profiles are in fair agreement following the trend predicted by the DNS.

Fig. 7 A nearly constant slope in semilog plot of error versus polynomial order signifies an exponential convergence. A polynomial order of 10 is used for the present study since an error close to $10^{-5}$ is achieved.
Preliminary simulations indicated an inflow angle deviation of about 2 deg at the inlet of the cascade; therefore the angle is chosen to match the cascade inlet angle by offsetting the inflow. The location of the cascade inlet is matched to the experiment and is at $x/C_{ax} = -0.16$.

### 3.4 Anisotropy Invariance Mapping

Lumley and Newman [28] quantified the anisotropy at a particular region of a flow by anisotropy invariance maps (AIMs). This is achieved by considering the second ($\Pi_2$) and third invariants ($\Pi_3$) of the anisotropy tensor, $a_{ij}$:

\begin{align}
\Pi_2 &= a_{ij}a_{ji} \\
\Pi_3 &= a_{ij}a_{jk}a_{ki} 
\end{align}

For two point and axisymmetric turbulence the relations are given by Eq. (14) and Eq. (15),

\begin{align}
\Pi_2 &= \frac{3}{2} \left( \frac{1}{3} \Pi_3 \right)^{2/3} \\
\Pi_3 &= \frac{2}{9} + 2\Pi_2
\end{align}

Plotting $\Pi_2$ against $\Pi_3$ maps any point in the flow to this bounded domain and turbulence is classified depending on its location in the plot. The limiting states are isotropic turbulence, one-component turbulence and isotropic two component turbulence. The lowermost point in the map represent isotropic turbulence where anisotropy is absent. Axisymmetric turbulence is found in both the branches of the map, the right side signifies the dominance of one fluctuating component as compared to the rest and the left one represents the least fluctuating velocity component [29]. The reader is referred to Lumley and Newman [28] for a detailed explanation on the representation of the invariant maps.

### 3.5 Time Orthogonal Decomposition

To further examine the physics and dynamics of the relaminarization bubble, the 3D time dependent data is analyzed using a technique known as Karhunen-Loève or proper orthogonal decomposition (POD). This decomposes a long-time data set into its most optimal basis functions ($\Phi_{X}$ - $\Phi_{X}$ basis function) accommodates a better treatment of the pressure terms.

The mesh is highly resolved near the suction and pressure surface with four elements clustered near the proximity of the blade and two elements inside the boundary layer. This corresponds to about 20 grid points inside the boundary layer, which is more than sufficient for resolving the boundary layer. Owing to the distribution of quadrature points in GLL quadrature, the first node is placed very close to wall. The $y^+$ near the separation zone is well below 1. A p-type refinement can be easily done without re-meshing and thus enables higher resolution for flows at high Reynolds number. Also from a computational perspective, the simulation can be restarted from a lower order run which enables considerable savings in CPU-hours during flow initialization. In total, $25 \times 10^6$ grid points are used for the present simulation.

Preliminary simulations indicated an inflow angle deviation of about 2 deg at the inlet of the cascade; therefore the angle at the inflow is offset by 2 deg compared to the experiments. The deviation could be attributed to the deflection of the flow just before the cascade. However, minor angle deviations at the inlet were observed experimentally but since no published data is available for the deviation, the angle is chosen to match the cascade inlet angle by offsetting the inflow. The location of the cascade inlet is matched to the experiment and is at $x/C_{ax} = -0.16$.

\[ a_{ij} = \frac{\nu \partial^2}{2k} - \frac{\delta_{ij}}{3} \]  
\[ \Pi_2 = a_{ij}a_{ji} \]  
\[ \Pi_3 = a_{ij}a_{jk}a_{ki} \]  

\[ \Pi_2 = \frac{3}{2} \left( \frac{1}{3} \Pi_3 \right)^{2/3} \]  
\[ \Pi_3 = \frac{2}{9} + 2\Pi_2 \]  

Here, $u, v$ and $w$ are the nondimensionalized velocity components in streamwise, pitchwise and spanwise direction, respectively. Although the signal vector $A_i$ is only the velocity vector, this generalization keeps for any amount of signals (adding temperature, concentration, etc.). The signal is decomposed into scalar spatial
functions \( \phi^k(x) \) and vector temporal function sets \( \Psi^k(t) \) for the \( k^{th} \) eigenfunction,

\[
A_k(x, t) = \sum_{k=0}^{\infty} \phi^k(x) \Psi^k(t)
\]  

(17)

These are computed by first solving an eigenvalue problem

\[
\int_0^T r_0(t, t') \Psi^k(t') dt' = (x^2) \Psi^k(t)
\]

(18)

\[
r_0(t, t') = \int \phi^k(x, t) A_k(x, t') dx
\]

(19)

where \( r \) stands for a spatial integral and \( \int_0^T \) is an integration over time. Then, projecting the time function set \( \Psi^k(t) \) onto the signal

\[
\int_0^T A_k(x, t) \Psi^k(t) dt = \phi^k(x)
\]

(20)

yields the spatial eigenfunctions \( \phi^k(x) \).

The spatial integration in Eq. (19) is numerically approximated by a summation over all control volumes, which are known from the meshed geometry. All time integrations are approximated using a trapezoidal quadrature, where the integration weight in dimensional time units, this corresponds to 7T, where \( T \) is defined as \( C_m/\alpha_u \). To analyze the time averaged separation bubble, velocity profiles at different locations on the suction side are considered. The spatially and temporally averaged streamwise velocity profiles on suction surface are presented in Fig. 11. The velocity profile at \( s/s_0 = 0.61 \) is inflectional and marks the beginning of separation. The boundary layer stays laminar as far as \( s/s_0 = 0.73 \) which can be evidenced from the instantaneous velocity profiles. The flow is altered downstream, which causes the reattachment at \( s/s_0 = 0.81 \).

The instantaneous velocity profile at four different time instants are shown in Fig. 12. At \( y/l = 0.015 \) the flow is steady and two dimensional with minimal change in spanwise velocity up to \( s/s_0 = 0.70 \). Further downstream the oscillations become violently unsteady and reach a maximum of about 40% of the freestream velocity at \( s/s_0 = 0.80 \) after which the oscillations seem to subside. However, the maximum streamwise reverse flow velocity, from Fig. 13, never reaches more than 15% of freestream, showing the presence of convective instability according to the classification by Almon and Sandham [1].

The separated shear layer is approximately at a distance of \( y/l = 0.045 \) from the suction surface. It is evident from Fig. 12, that the flow is not two dimensional and spanwise oscillations, although minor, occur even in the laminar regime. The oscillations propagate violently at \( s/s_0 = 0.85 \) reaching a maximum of over 30% of freestream velocity. This can be due to the unsteadiness associated with the reattachment. The frequency and the energy containing modes of the shear layer is presented in Sec. 4.4.

The velocity profiles shown in Fig. 14 compare the experiment and computed values along the suction side. The velocity profiles are in excellent agreement with the experiment in the laminar flow regime. However, minor differences in the boundary layer growth can be observed in the transitional and turbulent regime. It should also be noted that a single wire probe does not recognize the direction of the flow and the experimental data has not been mirrored in the plot.

The streamwise rms velocity profiles shown in Fig. 15 are in reasonable agreement with the experiment in the laminar regime but show under-prediction of the fluctuations inside the boundary layer. Also, in the turbulent regime, LES over-predicts the rms. The turbulent flow regime is highly unsteady with spanwise oscillations of more than 25% of the freestream velocity. This makes experimental prediction with a single wire probe in the streamwise direction extremely difficult and error-prone.

4 Results and Discussion

The time and spatially averaged pressure distribution around the blade is shown in Fig. 10. The pressure distribution gives a fair description of the flow phenomenon around the blade indicating the regions of separation, transition and reattachment. The pressure side is subjected to very minimal adverse pressure gradient at the leading edge and hence a small separation and reattachment is seen. Minor deviations between inlet flow and metal angle have also been reported in the experiments, which is the principle reason for the pressure side separation. However, the conditions on the suction side are much different with highly accelerated flow up to \( s/s_0 = 0.53 \), where \( c_p \) reaches a minimum. Further downstream, the flow is dominated by the adverse pressure gradient that causes the boundary layer to separate, at \( s/s_0 = 0.61 \). The flow is laminar as far as \( s/s_0 = 0.725 \) evidenced by a fairly constant pressure region, also known as dead air region. The separation bubble, in a time mean sense, extends from \( s/s_0 = 0.61 \) to \( s/s_0 = 0.81 \) occupying about 20% of the suction surface. Experimentally, the bubble occupies 23% of the suction surface from \( s/s_0 = 0.55 \) to \( s/s_0 = 0.78 \).

4.1 Statistical Quantities. Statistical quantities are monitored after allowing the initial transients to leave the domain. One flow-through time is defined as the average time required for a fluid particle to move from the inflow to the outflow plane. About four flow-through times are allowed for initial transients and data is subsequently monitored for one flow through time. In terms of non-dimensional time units, this corresponds to 7T, where \( T \) is defined as \( C_m/\alpha_u \). To analyze the time averaged separation bubble, velocity profiles at different locations on the suction side are considered. The spatially and temporally averaged streamwise velocity profiles on suction surface are presented in Fig. 11. The velocity profile at \( s/s_0 = 0.61 \) is inflectional and marks the beginning of separation. The boundary layer stays laminar as far as \( s/s_0 = 0.73 \) which can be evidenced from the instantaneous velocity profiles. The flow is altered downstream, which causes the reattachment at \( s/s_0 = 0.81 \).

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The streamwise rms velocity profiles shown in Fig. 15 are in reasonable agreement with the experiment in the laminar regime but show under-prediction of the fluctuations inside the boundary layer. Also, in the turbulent regime, LES over-predicts the rms. The turbulent flow regime is highly unsteady with spanwise oscillations of more than 25% of the freestream velocity. This makes experimental prediction with a single wire probe in the streamwise direction extremely difficult and error-prone.

4.2 Structure and Dynamics of Flow

4.2.1 Suction Surface. The flow structures prior and after separation and time evolution of vortex structures are visualized.
using the $\lambda_2$ criterion [40]. Figure 16 shows the spanwise vortices along the suction surface at different time instants. A minor spanwise disturbance sets in near $s/s_0 = 0.725$ that causes oscillatory motion of the separated shear layer as shown in Fig. 16(a). The manifestation of this disturbance causes vortex shedding downstream that resemble the hairpin structure shown in Fig. 16(b). The legs of the vortex are engulfed in the velocity deficit regions of boundary layer while the tip of the vortex is accelerated by the freestream. As a result of this shear, the vortex is subjected to stretching and the hairpin structures become elongated and highly skewed. This is followed by the entrainment of freestream into the boundary layer causing an enhancement in mixing (Fig. 16(c)).
ment in the laminar flow regime as far as the freestream velocity and the associated instability could be classified as convective according to Alam and Sandham [1].

The velocity profiles are in excellent agreement with the experiment in Horton [41] that consists of a dead-air zone and a separation bubble in Fig. 17 is identical to the classification structures in the freestream. However, the structures seem to be convoluted. Downstream of the trailing edge, a big hairpin vortex can be clearly seen which signifies that hairpin structures are elongated but carried by the freestream. The hairpin vortex structures have been observed by McAuliffe [3] among many others.

The AIM maps are plotted at different locations on the suction surface that describe the behavior of turbulence prior and after-separation and also inside and outside of the shear layer. At $s/s_o = 0.57$, the turbulence inside the shear layer exhibits two-point turbulence. This is typical as pointed out in Ref. [29] since the wall-normal components go to zero and wall parallel components only contribute to turbulence. A similar conclusion can be drawn at $s/s_o = 0.65$; however, here the turbulence beneath the shear layer slowly shifts away from the two-component turbulence and tends to move towards the isotropic two-component turbulence (two fluctuating components are equal). Near the region where the instability grows at $s/s_o = 0.73$, as one moves away from the wall inside the shear layer, turbulence transit to a nearly isotropic state from two-component turbulence state and as one moves beyond the shear layer, the turbulence exhibits an axisymmetric turbulence state (on the right branch of the map). This signifies the dominance of one component of turbulence over the other and from Fig. 21, we can see that this is due to spanwise fluctuations. Further downstream at $s/s_o = 0.85$, turbulence has completely shifted flow regimes between isotropic and two-component isotropic turbulence. The above AIM maps clearly show the highly unsteady nature of the transitional boundary layer, especially the different facets of turbulence during separation and reattachment.

4.2.2 Pressure Surface. Near the leading edge of the pressure surface, A vortices are formed, but since the flow accelerates, the structures do not persist for long. The head of A vortices split into two distinct streamwise structures and is elongated by the freestream. It then appears as long streamwise structures near the trailing edge as shown in Fig. 18.

4.3 Boundary Layer Parameters. Boundary layer parameters essential for characterizing the shape and growth of boundary layer are calculated. The average boundary layer edge velocity is calculated at different streamwise locations from which the displacement and momentum thickness are obtained. The displacement thickness, $\delta_1$ shows a nominal increase in the laminar regime and reaches a maximum at $s/s_o = 0.73$ when breakdown to turbulence occurs. At this point there is enhanced...
mixing and entrainment from the free stream which reduces the boundary layer thickness but increases the momentum thickness, $\delta_2$ as shown in Fig. 22. Upon reattachment the momentum thickness decreases. The shape factor (Fig. 22) exhibits a very similar trend until transition, but declines sharply in the turbulent regime of the flow reaching a value of two at the time mean reattachment point.

4.4 Time-Orthogonal Decomposition. The dynamics of the separation bubble on the blade suction side are examined using a variant of proper orthogonal decomposition based on Aubry et al. (1991) [38]. To center the data, the temporal mean is subtracted beforehand. Further, the analysis is limited to the separation region by setting all data upstream of $X/C_{x}=0.715$ to zero. The entire domain is then rotated around the Z-axis such that the U-component is aligned with the blade exit and the V-component is pointing in the wall normal direction with respect to the blade wall in the trailing region. 900 data samples where collected at a sampling rate of roughly 95 Hz.

This orthogonal decomposition guarantees optimality in a sense that the least number of modes $k$ (basis functions) is needed to reproduce a given fraction of the original signal’s energy. In this context, one can define the dimension of the system as the number of modes needed to capture 90% of the fluctuating kinetic flow energy, which is 1339 in this case. In an attempt to capture the dominant frequencies of shear layer flapping caused by the separation bubble and the vortex shedding therefrom, it is assumed that these flow features exhibit a harmonic oscillation in time. Thus, a

Fig. 16  Spanwise vorticity along the suction surface at different time instants. The contours are superimposed on isolines of spanwise vorticity. Blue indicates a negative value and red represents a positive value. (a), (b): The separated shear layer is unstable and sheds vortices downstream that appear like hairpin structures. (c): The growth of recirculation region beneath the shear layer can be seen. Circles indicate the physical phenomenon discussed.

Fig. 17  Velocity contours near the suction surface boundary layer at different time instants. The contours are superimposed on isolines of spanwise vorticity. (a)-(c): Flapping of the separated shear layer and formation of roll-up vortex is clearly seen. (c): The shear layer is stabilized possibly by the growth of the recirculation zone underneath. Circles indicate the physical phenomenon discussed.

Fig. 18  (a) Hairpin vortex structures can be seen near the leading edge of pressure surface, but due to the flow acceleration, gets elongated by the freestream and appear as long streamwise streaks. (b) Hairpin vortices near the trailing edge of the blade are marked with arrows. A nearly hairpin structure can be seen even downstream of the blade.
Fourier Transformation is performed on the time basis functions $\Psi_k(t)$, shown in Fig. 24. The first few modes contain very little dynamics in the streamwise ($U$) component, but some distinct frequency peaks at 0.65 Hz (for example mode 3) in the wall normal ($V$) part.

From the spatial representation in Fig. 25, top left, it can be seen that these high energy modes mostly capture free stream fluctuations in the middle of the passage. Due to the high complexity of the system (also indicated by the relatively high dimension), the time functions are not always best characterized by harmonic sine or cosine functions but show a rather erratic oscillation in some cases. Furthermore, the distribution in the homogeneous $z$-direction is not always even, which means that similar flow structure might be captured by different modes at different spanwise locations.

Figure 25 exemplarily shows representations of some typical mode classes which were found in the first 50 basis functions. The first ten modes are mostly active in the free stream with larger streamwise extent compared to the pitch and spanwise direction. Lower energy modes cover the separation region, where they show a smooth isosurface and contain relatively little structure in the vortex shedding and trailing area (for example, mode 28). Further down in the energy spectrum, the modes become more and more active downstream of the reattachment and extend further.
downstream of the trailing edge with relatively little activity in the separation region and free stream. The first mode with more or less periodic oscillation in time (streamwise component) and periodic structure downstream of the trailing edge is mode 30. Its dominant frequency is around 2.22 Hz in the \( U \) component, which is slightly above the expected vortex shedding frequency range of 1.83 Hz–2.14 Hz \[2\]. Mode 42 has a similar periodic spatial footprint, but its frequency peak in \( U \) is twice the frequency of mode 30. Thus, it might be considered the first harmonic.

In order to find the shear layer flapping frequency, one has to look for distinct and equal peaks in the wall normal (\( V \)) and streamwise (\( U \)) component in time as well as activity mostly in the separation region—without being active in the free stream, downstream of the blade and on the pressure side. The first mode fitting these criteria is mode 28, whose frequency peaks are at 0.42 Hz—which is within the experimental range of 0.37 Hz–0.61 Hz \[2\]. Another interesting mode with distinct peaks and thus harmonic oscillation is mode 34. The spatial representation resembles roll structures found in POD of turbulent channel DNS by Ball et al. (1991) \[32\]. The roll angle is roughly 45 deg with respect to the free stream on a plane formed by the streamwise and spanwise directions. The mode is most distinct over the blade surface and downstream of the reattachment region.

Fig. 21  Spanwise velocity fluctuations (a) and Reynolds stress (b) on the suction surface at \( s/s_0 = 0.49, 0.57, 0.61, 0.65, 0.73, 0.77, 0.85 \) with each profile offset to the right. Very minor spanwise fluctuations can be observed from \( s/s_0 = 0.49 \), however the growth is pronounced at \( s/s_0 = 0.77 \). This phenomenon near the reattachment region could be attributed to the highly unsteady boundary layer.

Fig. 22 (a) Variation of displacement and momentum thickness inside the boundary layer. The displacement thickness is maximum at \( s/s_0 = 0.73 \) and decreases further downstream which can be envisioned due to the transition to turbulence. (b) A similar trend can be seen for the form factor. The form factor reduces after \( s/s_0 = 0.73 \) that is typical of turbulent boundary layers.

downstream of the trailing edge with relatively little activity in the separation region and free stream. The first mode with more or less periodic oscillation in time (streamwise component) and periodic structure downstream of the trailing edge is mode 30. Its dominant frequency is around 2.22 Hz in the \( U \) component, which is slightly above the expected vortex shedding frequency range of 1.83 Hz–2.14 Hz \[2\]. Mode 42 has a similar periodic spatial footprint, but its frequency peak in \( U \) is twice the frequency of mode 30. Thus, it might be considered the first harmonic.

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quickly loses its structure downstream of the trailing edge. The wave speed is approximately 60% of the free stream. Ball et al. (1991) [32] showed that roll modes are the most energetic ones in canonical channel flow and are responsible for turbulent transport of fluid away from the walls, which is necessary for ‘bursts’ and ‘sweeps.’

5 Conclusions

The separation and reattachment of the suction surface boundary layer is characterized using large eddy simulations. On the suction side of the blade, at $s/s_{so} = 0.725$, minor spanwise disturbances start to occur. This instability propagates violently, introducing an oscillatory motion of the shear layer that causes A vortices to shed downstream. The breakdown of these hairpin vortices causes the transition to turbulence and thus the reattachment. This is in conjunction with the findings of Roberts et al. [5] and Yang and Voke [2]. The frequency ranges of shear layer flapping and vortex shedding are established using time orthogonal decomposition (TOD). The analysis also showed that modes capturing free stream fluctuations are the most energetic ones overall.

The pressure distribution on the suction side is in fair agreement with the experiment except near the separation regime. This can be attributed to the LES model. The streamwise extent of the separation bubble is under-predicted by 3% as compared to the experiment. The pressure distribution and velocity profiles follow precisely the trend predicted by the experiments.

Time-Orthogonal Decomposition of the flow data extracted the modes responsible for vortex shedding and shear layer flapping, identified by spatial and temporal dynamics. Some vortex shedding modes show roll-like regions of activity, angled $\sim 45$ deg with respect to the main flow. Such appearance is distinctive of turbulent POD modes seen in smooth channel flow, where they are responsible for large amounts of turbulent transport.

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Nomenclature

- $Re_{ss}$ = suction surface Reynolds number ($UC_{ax}/\nu$)
- $\delta_1$ = displacement thickness of a boundary layer
- $\delta_2$ = momentum thickness of a boundary layer
- $H_{12}$ = form factor
- $l$ = separation bubble length
- $C_{ax}$ = axial chord
- $C$ = chord
- $D$ = diameter of the hemisphere
- $C_{l,H}^{HPF}$ = high pass filtered Smagorinsky Model Constant
\[ Tu = \text{turbulence intensity} \]
\[ C_p = \text{pressure coefficient, } (P - P_\infty)/(1/2 \rho U_\infty^2) \]
\[ \rho = \text{density} \]
\[ s = \text{surface coordinate} \]
\[ s_o = \text{suction or pressure surface length} \]
\[ U_\infty = \text{freestream velocity} \]
\[ U = \text{boundary layer edge velocity} \]
\[ U_{ax} = \text{axial component of velocity, } (U_{ax} = U \cos \theta) \]
\[ \alpha = \text{angle of inlet velocity with horizontal } (-35 \text{ deg}) \]
\[ \kappa = \text{kinematic viscosity} \]
\[ \nu_t = \text{turbulent kinetic energy} \]
\[ \nu_f = \text{Eddy viscosity} \]
\[ \psi_{HPF} = \text{high pass filtered eddy viscosity} \]
\[ \tilde{u} = \text{high pass filtered velocity} \]
\[ \delta_k = \text{kronecker } \delta \]
\[ \psi = \text{stream function} \]
\[ \Delta = \text{filter width} \]
\[ S_{ij} = \text{strain tensor} \]
\[ \gamma = \text{mean rate of strain} \]
\[ \tau = \text{wall shear stress} \]
\[ \sigma = \text{shear velocity} \]
\[ \chi^+ = \text{sub-grid scale shear stress} \]
\[ T = \text{C_p U_\infty} \]
\[ \sigma_{ij} = \text{anisotropy tensor} \]
\[ \varphi_s(\tilde{x}) = \text{scalar spatial functions} \]
\[ \Psi_i(t) = \text{vector temporal function sets} \]
\[ II, III = \text{second and third invariant of anisotropy tensor} \]
\[ Re = \text{Reynolds Number} \]
\[ rns = \text{root mean square} \]
\[ AIM = \text{anisotropy invariance map} \]
\[ DNS = \text{direct numerical simulation} \]
\[ FSTI = \text{freestream turbulence intensity} \]
\[ GLSS = \text{gass-lobatto-legendre} \]
\[ HPF = \text{high pass filter} \]
\[ LES = \text{large eddy simulation} \]
\[ LPT = \text{low pressure turbine} \]
\[ POD = \text{proper orthogonal decomposition} \]
\[ TOD = \text{time orthogonal decomposition} \]
\[ RANS = \text{Reynolds averaged Navier-Stokes equation} \]
\[ SGS = \text{sub-grid scale} \]

**References**


